Pergamon

Int. J. Heat Mass Transfer. Vol. 38, No. 16, pp. 3065–3076, 1995

0017–9310/95 $9.50+0.00

0017–9310(95)00034–8

Hyperbolic axial dispersion model: concept and its application to a plate heat exchanger†

WILFRIED ROETZEL and SARIT K. DAS

Institute of Thermodynamics, University of the Federal Armed Forces Hamburg, D-22039 Hamburg, Germany

(*Received* 19 *September* 1994 *and in final form* 8 *December* 1994)

Abstract—A new concept of hyperbolic axial dispersion in fluid is introduced. This is an extension of the already established method of considering axial dispersion which takes the flow maldistribution into account in the analysis of heat exchangers. The concept is introduced by analogical treatment of the axial dispersion with the fluid conduction. Hyperbolic conduction, which considers a finite conduction wave propagation velocity, is important only in special cases such as cryogenic temperatures or sudden incidence of high heat flux. On the other hand the similar propagation velocity of the dispersion wave appears to be a general phenomenon which affects the thermal performance of heat exchangers even for common applications. Based on the proposed theoretical foundation, the dynamic analysis of a U-type plate heat exchanger is presented for step and sinusoidal change in one of the inlet temperatures. For this purpose the traditional inlet boundary condition for the dispersion model has been extended to incorporate the effect of the finite propagation velocity of the dispersion wave. The method of Laplace transforms has been applied for the analysis, and the Laplace inversion is carried out numerically using fast Fourier transforms. The results indicate that the proposed concept of 'hyperbolic dispersion' can be developed as a powerful tool for the analysis of heat exchangers particularly in the transient regime of operation.

INTRODUCTION

The steady state and transient behaviour of heat exchangers have been a constant topic of investigation for the heat-transfer community. With the many innovations in computational machinery the challenge of predicting the thermal behaviour of heat exchangers accurately and at the same time using simpler modelling concepts has been taken up in different ways by different investigators. An important contribution to this area is the method of introducing an axial dispersion term in the fluid energy equation which takes the deviation of the flow pattern from the plug flow model into consideration. A series of studies have been made in this regard [1–4] which justifies such an approach for both steady state and dynamic behaviour of heat exchangers. The axial dispersion is characterized by a dispersive Péclet number based on the dispersion coefficient λ^*. This coefficient can be regarded as a virtual thermal conductivity (of heat) in the fluid. The difference between dispersion and real conduction is that the dispersion coefficient is higher in order of magnitude and it is a flow property rather than a fluid property.

Since the dispersion phenomenon is visualized as a virtual conduction in fluid, its behaviour can be understood by analogical treatment with thermal conduction in fluid. In the absence of heat sources and with constant thermal conductivity, heat conduction is usually described by the parabolic equation

$$\frac{1}{\alpha}\frac{\partial T}{\partial \tau} = \nabla^2 T \tag{1}$$

which is valid under the definition of λ in Fourier heat conduction as

$$\dot{q} = -\lambda \nabla T. \tag{2}$$

Equation (2) is empirical in nature which assumes an infinite propagation velocity of the thermal wave and is valid only for 'slower' changes in temperature as a good approximation. However, for special cases such as cryogenic temperatures or very fast change of temperature because of the sudden incidence of a high heat flux, equation (1) is to be replaced by the hyperbolic or so-called 'non-Fourier' conduction equation [5, 6] in the form

$$\frac{1}{C^2}\frac{\partial^2 T}{\partial \tau^2} + \frac{1}{\alpha}\frac{\partial T}{\partial \tau} = \nabla^2 T. \tag{3}$$

This equation presumes the heat conduction law proposed by Chester [7] as

$$\dot{q} + \frac{\alpha}{C^2}\frac{\partial \dot{q}}{\partial \tau} = -\lambda \nabla T. \tag{4}$$

This equation is not totally empirical and it stands partially on the foundation of theoretical analysis which shows it to be a better approximation for the special cases mentioned above. In recent times there

†Dedicated to Prof. Dr.-Ing, Dr.-Ing, E.h. Karl Stephan on his 65th birthday.

NOMENCLATURE

- A effective heat transfer area per plate, m^2
- $\mathbf{A}$ coefficient matrix for system of differential equations
- A_c free flow area in a channel, m^2
- $A_{k,m}$ the element of kth row and mth column of matrix $\mathbf{A}$
- b plate width, m
- $\mathbf{B}$ diagonal matrix, equation (61)
- C conduction wave propagation velocity, $m\ s^{-1}$
- C^* dispersion wave propagation velocity, $m\ s^{-1}$
- c_p isobaric specific heat of fluid, $J\ kg^{-1}\ K^{-1}$
- $\mathbf{D}$ matrix resulting from boundary conditions, equation (67)
- d_j elements of matrix $\mathbf{D}$
- $f(Z)$ inlet temperature function
- F wetted perimeter of the flow channel, m
- $F(s)$ Laplace transform of $f(Z)$
- $\mathbf{F}$ matrix with inlet fluid temperature functions, equation (73)
- $g_{i,j}$ elements of matrix $\mathbf{G}$
- $\mathbf{G}$ matrix of eigenvectors of coefficient matrix $\mathbf{A}$
- h heat transfer coefficient, $W\ m^{-2}\ K^{-1}$
- $\bar{h}$ enthalpy of the fluid, $J\ kg^{-1}$
- i square root of -1
- l_i path traversed by fluid particle before entering ith channel, m
- L fluid flow length in channels
- m_j $= j-2[j/2]$, where j is an integer
- n number of channels on one side
- N total number of channels
- Pe axial dispersive Péclet number, $\dot{w}L/A_c\lambda^*$
- Pe_C effective Péclet number $= Pe/(1-V^2)$
- $\dot{q}$ heat flux, $W\ m^{-2}$
- $\dot{q}_x$ axial heat flux in fluid, $W\ m^{-2}$
- $\dot{q}_w$ wall heat flux, $W\ m^{-2}$
- $\dot{Q}$ axial heat flow rate in the fluid, W
- R_2 capacity rate ratio in the channels, $\dot{w}_2/\dot{w}_1$
- R_c ratio V_2/V_1
- R_{g2} capacity rate ratio of the combined flow, $\dot{w}_{g2}/\dot{w}_{g1}$
- R_{gu} velocity ratio of the combined fluid, u_{g2}/u_{g1}
- R_N ratio U_2/U_1
- R_{Pe} ratio of Péclet numbers in channels, Pe_2/Pe_1
- R_u velocity ratio, u_2/u_1
- R_w wall heat capacity ratio, W_w/W_1
- S transformed time variable in Laplace domain
- t temperature obtained by Laplace transformation of dimensionless temperature Θ
- T temperature, K
- $\mathbf{T}$ temperature matrix, equation (60)
- u fluid velocity, $m\ s^{-1}$
- $u_{i'}$ fluid velocity in the gasket port after ith channel, $m\ s^{-1}$
- u_g fluid velocity of the combined flow, $m\ s^{-1}$
- $U_{1(2)}$ $= (hA/\dot{w})_{1(2)}$
- V ratio of the fluid velocity to the dispersion wave velocity, u/C^*
- $\dot{w}$ thermal capacity rate of fluid in one channel, $W\ K^{-1}$
- $\dot{w}_g$ thermal capacity rate of the combined fluid, $W\ K^{-1}$
- W heat capacity of fluid(s), $J\ K^{-1}$
- $\mathbf{W}$ matrix defined by equation (65)
- W_w heat capacity of wall, $J\ K^{-1}$
- x dimensionless space coordinate, X/L
- X space coordinate, m
- $Y_1, Y_2 \ldots Y_6$ coefficients of equation (40)
- Z dimensionless time τ/τ_{r1}.

Greek symbols

- α thermal diffusivity, $m^2\ s^{-1}$
- α^* thermal diffusivity of axial dispersion $= \lambda^*/\rho c_p$, $m^2\ s^{-1}$
- β_j jth eigenvalue of matrix $\mathbf{A}$
- $\gamma_1, \gamma_2, \gamma_3, \gamma_4$ coefficients, equations (41)–(44)
- γ_c, γ_u coefficients, equations (45)–(46)
- Γ a unit step function
- Θ dimensionless temperature $= T-T_{g1,in}/T_{g2,in}-T_{g1,in}$
- λ thermal conductivity, $W\ m^{-1}\ K^{-1}$
- λ^* axial dispersion coefficient, $W\ m^{-1}\ K^{-1}$
- ρ fluid density, $kg\ m^{-2}$
- τ time, s
- τ_r residence time, $W/\dot{w}$
- ϕ dimensionless phase lag (cumulative value)
- $\Delta\phi$ dimensionless phase lag (discrete value).

Subscripts

- exit at exit
- g combined flow before splitting in channels or after recombination at exit
- i ith channel
- in at inlet
- w plate/wall
- wi ith plate
- 0 initial
- 1 the fluid in odd channels
- 2 the fluid in even channels
- $+$ the section just behind the heat exchanger inlet where dispersion begins
- $-$ the section just in front of the heat exchanger inlet where the dispersion begins.

has been a number of investigations [8, 9] dealing with this hyperbolic heat conduction problem.

However, apart from the special cases mentioned above, generally in all usual cases of technical interest the term α/C^2 in equation (4) is negligible because of the very high propagation velocity C of the thermal wave. The case with axial dispersion appears to be different in this respect. In dispersion the apparent thermal diffusivity due to fluid mixing is of much higher order of magnitude than with pure conduction or mass diffusion. On the other hand the propagation velocity of thermal disturbance caused by real or virtual axial mixing is much smaller. It may be even of the order of the flow velocity under consideration. Therefore the infinite propagation velocity of the dispersion wave assumed so far [1–4] does not depict the real picture of heat transfer. It can be at best taken as the best approximation available so far.

Recently, an extended axial dispersion model with hyperbolic conduction or dispersion has been derived and applied to the special case of flow in an adiabatic channel [10, 11]. It has been shown that a finite propagation velocity may have a remarkable effect on the outlet response to a step change in inlet temperature.

In the present work the hyperbolic axial dispersion model is further extended and applied to a U-type plate heat exchanger as an example. The sudden acceleration in the application and investigation [12, 13] of plate heat exchangers in recent times has acted as an inspiration to apply the proposed theory to this apparatus. The results are worth comparing with the recent analyses [14, 15] in order to bring out the significance of the present proposition.

THE HYPERBOLIC DISPERSION MODEL

In order to propose the model, first the heat conduction in the stationary fluid inside a pipe of constant cross-section has been considered. The fluid inside the pipe may be looked upon as a thin rod with axial heat conduction (or dispersion) and heat transfer from the stationary fluid to the stationary wall of changing temperature. For constant fluid properties the elemental energy balance in the fluid yields

$$\rho c_p \frac{\partial T}{\partial \tau} = -\frac{\partial \dot{q}_x}{\partial x} + \frac{F}{A_C}\dot{q}_w \tag{5}$$

where T is the mean temperature in the cross-section at location x and time τ. This temperature is the true fluid temperature if the radial thermal conductivity is infinitely large or if the radius of the pipe (rod) shrinks to zero. Two heat fluxes appear in equation (5): the conductive or dispersive axial heat flux $\dot{q}_x$ and the convective radial heat flux $\dot{q}_w$ at the inner wall surface.

Differentiating with respect to τ and multiplying the equation (5) with α/C^2 and then adding it to the equation (5) gives

$$\frac{\lambda}{C^2}\frac{\partial^2 T}{\partial \tau^2} + \rho c_p \frac{\partial T}{\partial \tau} = -\frac{\partial}{\partial x}\left(\dot{q}_x + \frac{\alpha}{C^2}\frac{\partial \dot{q}_x}{\partial \tau}\right) + \frac{F}{A_C}\left(\dot{q}_w + \frac{\alpha}{C^2}\frac{\partial \dot{q}_w}{\partial \tau}\right). \tag{6}$$

Under the law of hyperbolic conduction proposed by Chester as per equation (4), this equation (6) reduces to the general one-dimensional hyperbolic heat conduction equation with heat transfer from the periphery in the form

$$\frac{1}{C^2}\frac{\partial^2 T}{\partial \tau^2} + \frac{1}{\alpha}\frac{\partial T}{\partial \tau} = \frac{\partial^2 T}{\partial x^2} + \frac{F}{\lambda A_C}\left(\dot{q}_w + \frac{\alpha}{C^2}\frac{\partial \dot{q}_w}{\partial \tau}\right). \tag{7}$$

When the fluid rod inside the stationary tube moves with a constant velocity of u in the x-direction, we get plug flow model with constant free flow area and fluid density. Under such conditions the time derivatives of equation (7) should be replaced by the substantial differential operators

$$\frac{D}{D\tau} = \frac{\partial}{\partial \tau} + u\frac{\partial}{\partial x} \tag{8a}$$

and

$$\frac{D^2}{D\tau^2} = \frac{\partial^2}{\partial \tau^2} + 2u\frac{\partial^2}{\partial \tau \partial x} + u^2\frac{\partial^2}{\partial x^2}. \tag{8b}$$

Substituting equation (8) into equation (7) and using the dispersion coefficient λ^* and dispersion wave velocity C^* instead of the thermal conductivity λ and conduction wave velocity C gives

$$\frac{1}{C^{*2}}\left[\frac{\partial^2 T}{\partial \tau^2} + 2u\frac{\partial^2 T}{\partial \tau \partial x} + u^2\frac{\partial^2 T}{\partial x^2}\right] + \frac{1}{\alpha^*}\left[\frac{\partial T}{\partial \tau} + u\frac{\partial T}{\partial x}\right] = \frac{\partial^2 T}{\partial x^2} + \frac{F}{\lambda^* A_C}\left[\dot{q}_w + \frac{\alpha^*}{C^{*2}}\frac{D\dot{q}_w}{D\tau}\right]. \tag{9}$$

The last term of equation (9) can be treated in two different ways as described below.

(i) *Regenerator model*

The so-called 'regenerator model' [10] is applicable to the case where the fluid receives only one heat flux from a unique periphery. Under such conditions the heat transfer coefficient, in this model, is defined by

$$\dot{q}_w + \frac{\alpha}{C^2}\frac{D\dot{q}_w}{D\tau} = h(T_w - T). \tag{10}$$

This definition presumes that the peripherial heat flux encounters a delay effect similar to that experienced by the axial heat flux in fluid described by equation (4). This, in principle, is physically meaningful because heat transfer in such a case can also be considered as heat conduction through the fluid layer adjacent to the wall. However, the value of α/C^2 in such conduction will be smaller than the corresponding value by axial dispersion. Thus when equation (10) is used with α^*/C^{*2} values for axial dispersion the delay effect at the wall is overestimated.

When equation (10) is used in equation (9) a simpler form of energy equation for fluid can be obtained. The resulting energy equation for the wall, however, neglects axial conduction to be reduced to a simpler form. Furthermore, the regenerator model cannot be used for recuperators because the ratio α^*/C^{*2} of the fluid appears in the energy equation of the wall. If this value differs for the two fluids, no logical value for this parameters can be suggested for the wall equation since the wall is wetted by both the fluids. Therefore a different and even more realistic approach is resorted to for recuperators, which is also applicable to regenerators and thus generally valid for heat exchangers [11].

(ii) *General heat exchanger model*

In this model the usual definition of the heat transfer coefficient h is substituted in equation (9) as

$$\dot{q}_{\rm w} = h(T_{\rm w}-T) \tag{11}$$

This means that the delay effect at the wall, which is much smaller than the dispersion delay effect, has been neglected.

Substituting, equation (11) in energy equation (9) under the condition of constant heat transfer coefficient results in the hyperbolic energy equation for a fluid stream in the channel

$$\frac{1}{C^{*2}}\left[\frac{\partial^2 T}{\partial \tau^2}+2u\frac{\partial^2 T}{\partial \tau\,\partial x}+u^2\frac{\partial^2 T}{\partial x^2}\right]+\frac{1}{\alpha^*}\left[\frac{\partial T}{\partial \tau}+u\frac{\partial T}{\partial x}\right] = \frac{\partial^2 T}{\partial x^2}+\sum\frac{F}{\lambda^* A_C}\left[h(T_{\rm w}-T)+\frac{\alpha^* h}{C^{*2}}\times\left(\frac{\partial(T_{\rm w}-T)}{\partial \tau}+u\frac{\partial(T_{\rm w}-T)}{\partial x}\right)\right]. \tag{12}$$

The summation in the heat transfer term of this equation indicates that there can be more than one heat flux from the walls (as in the case of plate heat exchangers). The same equation with $u = 0$ and C^*, α^*, λ^* and A_C replaced by $C_{\rm w}$, $\alpha_{\rm w}$, $\lambda_{\rm w}$ and $A_{C{\rm w}}$ respectively, can be used as the wall energy equation. The conductive propagation velocity could be assumed to be infinitely high: $C_{\rm w} = \infty$. In many practical cases the wall longitudinal conduction can be entirely neglected, consequently the wall equation reduces to the well-known form

$$\frac{\partial T_{\rm w}}{\partial \tau} = \sum\frac{hF}{\rho_{\rm w} c_{p{\rm w}} A_{C{\rm w}}}(T-T_{\rm w}). \tag{13}$$

BOUNDARY CONDITIONS

In the traditional dispersion model ($C^* = \infty$) the boundary conditions of Danckwert [16] are valid. At the inlet to the exchanger a temperature drop $(T_{\rm in}^{(-)}-T_{\rm in}^{(+)})$ is present and at the outlet the temperature slope is zero $\partial T/\partial x/_{\rm out} = 0$. However, with finite values of C^* the inlet condition of Danckwert [16] is no longer valid and has to be extended to take finite propagation velocity into account. For the following derivation it is assumed that in the flow channel carrying fluid to the exchanger no dispersion takes place and that the inside surface of this channel is adiabatic.

At the inlet to the exchanger the energy balance yields

$$\dot{Q}_{\rm in}^{(-)}+\dot{m}\bar{h}_{\rm in}^{(-)} = \dot{Q}_{\rm in}^{(+)}+\dot{m}\bar{h}_{\rm in}^{(+)} \tag{14}$$

where $(-)$ and $(+)$ indicate the positions just in front $(-)$ of the inlet cross-section and just behind $(+)$ the inlet cross-section A_C of the exchanger. $\dot{Q}_{\rm in}$ are the conductive or dispersive axial heat flows in the fluid. With zero dispersion in front of the inlet $\dot{Q}_{\rm in}^{(-)} = 0$ and

$$\dot{Q}_{\rm in}^{(+)} = \dot{m}(\bar{h}_{\rm in}^{(-)}-\bar{h}_{\rm in}^{(+)}) = \dot{m}c_{\rm p}(T_{\rm in}^{(-)}-T_{\rm in}^{(+)}) \tag{15}$$

with $c_{\rm p}$ as the appropriate mean value between $T_{\rm in}^{(-)}$ and $T_{\rm in}^{(+)}$. Dividing by the cross-sectional area at the inlet and the exchanger channel A_C yields

$$\dot{q}_{x,{\rm in}}^{(+)} = u\rho c_{\rm p}(T_{\rm in}^{(-)}-T_{\rm in}^{(+)}). \tag{16}$$

Forming the substantial derivative, multiplying by α^*/C^* and adding to equation (16) yields with Chester's definition equation (4)

$$-\lambda^*\frac{\partial T_{\rm in}^{(+)}}{\partial x} = u\rho c_{\rm p}(T_{\rm in}^{(-)}-T_{\rm in}^{(+)}) + \frac{u\lambda^*}{C^{*2}}\frac{{\rm D}(T_{\rm in}^{(-)}-T_{\rm in}^{(+)})}{{\rm D}\tau}. \tag{17}$$

In the adiabatic channel to the exchanger inlet

$$\frac{{\rm D}T_{\rm in}^{(-)}}{{\rm D}\tau} = 0 \tag{18}$$

and equation (17) turns finally to

$$T_{\rm in}^{(-)}-T_{\rm in}^{(+)} = -\frac{\alpha^*}{u}\left(1-\frac{u^2}{C^{*2}}\right)\frac{\partial T_{\rm in}^{(+)}}{\partial x}+\frac{\alpha^*}{C^*}\frac{\partial T_{\rm in}^{(+)}}{\partial \tau}. \tag{19}$$

This new extended inlet boundary condition becomes the well-known Danckwert condition when $C^* \to \infty$.

The outlet condition remains the same as in the case of infinite value of C^*.

APPLICATION TO PLATE HEAT EXCHANGERS

The hyperbolic modelling concept for fluid axial dispersion developed in the preceding sections can be applied to a single-pass U-type plate heat exchanger to evaluate its transient responses. This will not only be an example of the significance of the present proposition but also an important extension of the analysis [14] presented earlier.

Mathematical formulation

For mathematical modelling of plate exchangers it is essential to resort to some assumptions. The assumptions are similar in nature to those in the previous study [14] but some simplifications are suggested which conform to the reality. These assumptions are

(1) All thermal properties are independent of temperature and pressure.
(2) The flow velocity and heat transfer coefficient are identical within the channels carrying similar fluids but they may be different for the two fluids.
(3) The thermal resistances of the plates are negligible across the width of the plates but they are infinite along the plate length. This means that longitudinal conduction in plates is neglected here and equation (13) is valid.
(4) Heat transfer takes place only across the plates and not through sealing edges or gaskets.
(5) The heat exchanger is thermally insulated from the atmosphere.
(6) The exchanger is started from the cold state (i.e. a uniform temperature).
(7) The flow is completely mixed in the transverse direction within a channel. This gives a uniform temperature at each single-channel cross-section.
(8) The flow maldistribution in flow passages can be described by introducing an axial dispersion term in the energy equation and this dispersion wave propagates with a finite velocity.
(9) The dispersion and heat transfer in the fluids starts at the entry to the channels and not in the port carrying fluid to the channels.

The channel nomenclature selected for the present heat exchanger has been shown in Fig. 1. The coordinate system is chosen in the direction of the cold fluid which always remains in the odd channels and the even channels carry the fluid where the temperature is changed at the entry. The channels are named from 1 to N and plates 1 to $N+1$. The figure is constructed for an odd number of channels, for even number of channels the fluid and its flow direction in the last channel has to be the opposite of that in this figure. With this nomenclature using the 'general heat exchangers model' of hyperbolic dispersion equation (12), the energy equation for the channel fluids can be written in the form

$$\frac{1}{C_i^{*2}}\left[\frac{\partial^2 T_i}{\partial \tau^2}+2u_i(-1)^{i-1}\frac{\partial^2 T_i}{\partial \tau\,\partial X}+u_i^2\frac{\partial^2 T_i}{\partial X^2}\right]+\left(\frac{1}{\alpha^*}+\frac{2\alpha^* hb}{C^{*2}\lambda^* A_C}\right)_i\left[\frac{\partial T_i}{\partial \tau}+(-1)^{i-1}u_i\frac{\partial T_i}{\partial X}\right]$$
$$=\frac{\partial^2 T_i}{\partial X^2}+\frac{h_i b}{\lambda_i^* A_{Ci}}(T_{wi}+T_{wi+1}-2T_i)+\frac{\alpha_i^* h_i b_i}{C_i^{*2}\lambda_i^* A_{Ci}}\left[\left(\frac{\partial T_{wi}}{\partial \tau}+\frac{\partial T_{wi+1}}{\partial \tau}\right)+(-1)^{i-1}u_i\left(\frac{\partial T_{wi}}{\partial X}+\frac{\partial T_{wi+1}}{\partial X}\right)\right]\quad (i=1,2,\ldots N). \tag{20}$$

The simplified energy equation for walls in the absence of longitudinal conduction can be written from equation (13) as

$$\frac{W_w}{L}\frac{\partial T_{wi}}{\partial \tau}=\frac{(hA)_1}{2L}(T_{i-1}-T_{wi})+\frac{(hA)_2}{2L}(T_i-T_{wi})\quad \left[i=2,4,6\ldots 2\left(\frac{N+1}{2}\right)\right] \tag{21}$$

$$\frac{W_w}{L}\frac{\partial T_{wi}}{\partial \tau}=\frac{(hA)_2}{2L}(T_{i-1}-T_{wi})+\frac{(hA)_1}{2L}(T_i-T_{wi})\quad \left[i=3,5,7\ldots 2\left(\frac{N}{2}\right)-1\right] \tag{22}$$

$$\frac{W_w}{L}\frac{\partial T_{w1}}{\partial \tau}=\frac{(hA)_1}{2L}(T_1-T_{w1}) \tag{23}$$

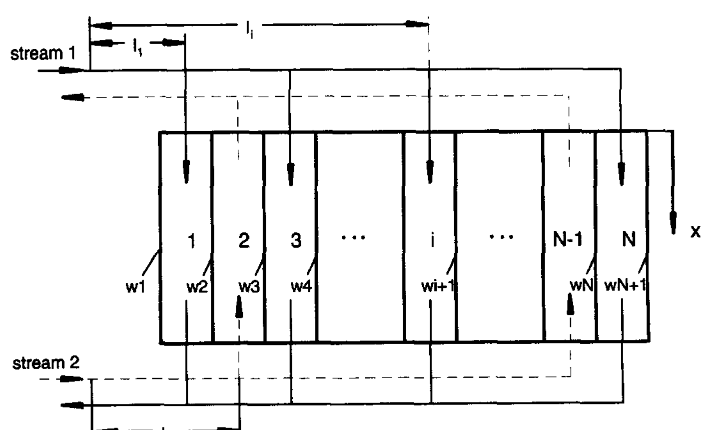

Fig. 1. Channel and plate nomenclature; w1, w2 ... wN+1 indicate the N+1 plates and 1, 2, ... N the N channels (odd number of channels assumed).

$$\frac{W_w}{L}\frac{\partial T_{WN+1}}{\partial \tau} = \frac{(hA)_N}{2L}(T_N - T_{WN+1}) \quad (24)$$

where $(hA)_N = (hA)_1$ for odd N

$= (hA)_2$ for even N.

These equations can be reduced to the non-dimensional form using the dimensionless groupings used as heat exchanger characteristics such as the number of transfer units NTU, the heat capacity rate ratio R_2 and also the newly introduced parameters such as the dispersive Péclet number Pe and the ratio V of the fluid to dispersion wave velocity. The set of non-dimensional variables chosen is

$$\tau_{r1} = \frac{W_1}{\dot{w}_1}, \quad \tau_{r2} = \frac{W_2}{\dot{w}_2}$$

$$U_1 = \frac{(hA)_1}{\dot{w}_1}, \quad U_2 = \frac{(hA)_2}{\dot{w}_2}$$

$$NTU_1 = \left[\frac{1}{U_1} + \frac{n_1}{n_2}\cdot\frac{1}{R_2 U_2}\right]^{-1}$$

$$R_2 = \frac{\dot{w}_2}{\dot{w}_1}, \quad R_w = \frac{W_w}{W_1}, \quad R_N = \frac{U_2}{U_1}$$

$$Pe_1 = \frac{\dot{w}_1 L}{A_C \lambda_1^*}, \quad Pe_2 = \frac{\dot{w}_2 L}{A_C \lambda_2^*}, \quad R_{pe} = \frac{Pe_2}{Pe_1}$$

$$V_i = \frac{W_i}{C_i^*}, \quad R_u = \frac{u_2}{u_1}, \quad R_C = \frac{V_2}{V_1}$$

$$x = X/L, \quad Z = \tau/\tau_{r1}, \quad \Theta = \frac{T - T_{g1,in}}{T_{g2,in} - T_{g1,in}}. \quad (25)$$

With the help of these dimensionless parameters equations (21)–(24) can be reduced to the following dimensionless forms:

$$V_1^2 R_C^{2m_{i+1}}\left[\frac{1}{R_u^{2m_{i+1}}}\frac{\partial^2\Theta_i}{\partial Z^2} + \frac{2(-1)^{i-1}}{R_u^{m_{i+1}}}\frac{\partial^2\Theta_i}{\partial x^2}\right]$$
$$+[Pe_1 R_{pe}^{m_{i+1}} + U_1 V_1^2 (R_N R_C^2)^{m_{i+1}}]$$
$$\times\left[\frac{1}{R_u^{m_{i+1}}}\frac{\partial\Theta_i}{\partial Z} + (-1)^{i-1}\frac{\partial\Theta_i}{\partial x}\right]$$
$$= \frac{\partial^2\Theta_i}{\partial x^2} + \frac{Pe_1 U_1}{2}(R_{pe}R_N)^{m_{i+1}}[\Theta_{wi} + \Theta_{wi+1} - 2\Theta_i]$$
$$+\frac{U_1 V_1^2}{2}(R_N R_C^2)^{m_{i+1}}\left[\frac{1}{R_u^{m_{i+1}}}\left\{\frac{\partial\Theta_{wi}}{\partial Z} + \frac{\partial\Theta_{wi+1}}{\partial Z}\right\}\right.$$
$$\left.+(-1)^{i-1}\left\{\frac{\partial\Theta_{wi}}{\partial x} + \frac{\partial\Theta_{wi+1}}{\partial x}\right\}\right] \quad (26)$$

$$R_w\frac{\partial\Theta_{wi}}{\partial Z} = \frac{U_1}{2}(R_N R_2)^{m_i}(\Theta_{i-1} - \Theta_{wi})$$
$$+\frac{U_1}{2}(R_N R_2)^{m_{i+1}}(t_i - t_{wi}) \quad (i = 2, 3, \ldots N) \quad (27)$$

$$R_w\frac{\partial\Theta_{w1}}{\partial Z} = \frac{U_1}{2}(\Theta_1 - \Theta_{w1}) \quad (28)$$

$$R_w\frac{\partial\Theta_{wN+1}}{\partial Z} = \frac{U_1}{2}(R_N R_2)^{m_{N+1}}(\Theta_N - \Theta_{wN+1}) \quad (29)$$

$$\text{where} \quad m_j = j - 2\left(\frac{j}{2}\right).$$

The Péclet numbers appearing in these equations indicate values within the channels and not of the combined flow before separation in the conduit carrying the fluid. The different ratios also indicate the values in channels given by

$$R_2 = R_{g2}\left(\frac{n_1}{n_2}\right)$$

$$R_u = R_{gu}\left(\frac{n_1}{n_2}\right).$$

The boundary conditions to equations (26)–(29) can be set considering the 'phase lag effect' introduced in a previous analysis [14]. This takes into account the fact that the fluid, even though given a unique temperature function at the inlet, enters the different channels with different phase lags ϕ_i. This is due to the fact that the fluid particles travel an increasing length of path before entering channels 1, 2, 3 ... N respectively. Consequently the velocity in the conduit can also be considered to be changing as per the continuity condition, as

$$\frac{u_{2i-1}}{u_{g1}} = 1 - \frac{i}{n_1} \quad (i = 1, 2, 3 \ldots n_1) \quad (30)$$

$$\frac{u_{2i'}}{u_{g1}} = 1 - \frac{i}{n_2} \quad (i = 1, 2, 3 \ldots n_2). \quad (31)$$

From these velocities the phase lag between the channels can be calculated as

$$\Delta\phi_1 = l_1/V_{g1}\tau_{r1} \quad (32)$$

$$\Delta\phi_2 = l_2/V_{g2}\tau_{r1} \quad (33)$$

$$\Delta\phi_{2i+1} = (l_{2i+1} - l_{2i-1})/V_{(2i-1)'}\tau_{r1}$$
$$(i = 1, 2, 3 \ldots n_1 - 1) \quad (34)$$

$$\Delta\phi_{2i-2} = (l_{2i+2} - l_{2i})/V_{2i'}\tau_{r1} \quad (i = 1, 2, 3 \ldots n_2 - 1). \quad (35)$$

The distances l_i are defined by Fig. 1. The total phase lag at the entry of each channel can now be calculated as

$$\phi_{2i-1} = \sum_{j=1}^{2i-1}\Delta\phi_{2j-1} \quad (i = 1, 2, 3 \ldots n_1) \quad (36)$$

$$\phi_{2i} = \sum_{j=1}^{2i}\Delta\phi_{2j} \quad (i = 1, 2, 3 \ldots n_2). \quad (37)$$

This phase lag effect is multiplied in a U-type plate exchanger where the fluid undergoes an identical phase lag at the exit as well, this is given by

$$\phi_{i,\text{exit}} = \phi_i. \quad (38)$$

The boundary conditions as discussed with equations (14) to (19) can be written in dimensionless form with the newly defined apparent Péclet number as

$$Pe_C = Pe/(1-V^2). \quad (39)$$

The boundary conditions reduce to:

at $x = 0$

$$\Theta_i - \frac{1}{Pe_{C1}}\frac{\partial \Theta_i}{\partial x} + \frac{V_1^2}{Pe_1}\frac{\partial \Theta_i}{\partial Z} = f_1(Z-\phi_i)\Gamma(Z-\phi_i)$$

$$\left[i = 1,3,5,\ldots,2\left(\frac{N+1}{2}\right)-1\right] \quad (40)$$

$$\frac{\partial \Theta_i}{\partial x} = 0 \quad \left[i = 2,4,6,\ldots 2\left(\frac{N}{2}\right)\right] \quad (41)$$

at $x = 1$

$$\Theta_i + \frac{1}{Pe_{C2}}\frac{\partial \Theta_i}{\partial x} + \frac{V_2^2}{Pe_2 R_u}\frac{\partial \Theta_i}{\partial Z} = f_2(Z-\phi_i)\Gamma(Z-\phi_i)$$

$$\left[i = 2,4,6,\ldots,2\left(\frac{N}{2}\right)\right] \quad (42)$$

$$\frac{\partial \Theta_i}{\partial x} = 0 \quad \left[i = 1,3,5,\ldots 2\left(\frac{N+1}{2}\right)-1\right]. \quad (43)$$

Solution for temperature response. The partial differential equations (26)–(29) along with the boundary conditions given by equations (40)–(43) can be solved by the method of Laplace transforms. For this, as per assumption (6) of the previous section, the initial conditions of fluid and wall are set to zero,

$$\Theta_{i,0} = \Theta_{wi,0} = 0.$$

Taking the Laplace transform of the wall equations (27)–(29) we get

$$t_{wi} = \frac{(U1/2)(R_N R_2)^{m_i} t_{i-1} + (U1/2)(R_N R_2)^{m_{i+1}} t_i}{SR_w + (U1/2)(R_N R_2)^{m_i} + (U1/2)(R_N R_2)^{m_{i+1}}}$$

$$(i = 2,3,\ldots N) \quad (44)$$

$$t_{wi} = \frac{(U1/2)t_1}{SR_w + U1/2} \quad (45)$$

$$t_{wN+1} = \frac{(U1/2)(R_N R_2)^{m_{N+1}} t_N}{R_w S + (U1/2)(R_N R_2)^{m_{N+1}}}. \quad (46)$$

Now, the Laplace transform of the channel equation (26) can be taken and in it the value of t_{w1}, $t_{w2}\ldots t_{WN+1}$ can be substituted from equations (44)–(46). Thus a system of ordinary differential equations is obtained in the form

$$\frac{d^2 t_i}{dx^2} = \frac{1}{1-\gamma_C}\left[Y_1 t_i + Y_2 t_{i-1} + Y_3 t_{i+1} + Y_4 \frac{dt_i}{dx} + Y_5 \frac{dt_{i-1}}{dx} + Y_6 \frac{dt_{i+1}}{dx}\right] \quad i = 1,2,3,\ldots,N \quad (47)$$

where

$$R_j = \frac{U_1}{2}(R_N R_2)^{m_j}$$

$$\gamma_1 = \frac{R_i}{SR_w + R_i + R_{i+1}} \quad i = 2,3,\ldots,N$$

$$= 0 \quad i = 1 \quad (48)$$

$$\gamma_2 = \frac{R_{i+1}}{SR_w + R_i + R_{i+1}} \quad i = 2,3,\ldots,N$$

$$= \frac{R_{i+1}}{SR_w + R_{i+1}} \quad i = 1 \quad (49)$$

$$\gamma_3 = \frac{R_{i+1}}{SR_w + R_{i+1} + R_{i+2}} \quad i = 1,2,\ldots,N-1$$

$$= \frac{R_{N+1}}{R_w S + R_{N+1}} \quad i = N \quad (50)$$

$$\gamma_4 = \frac{R_{i+2}}{SR_w + R_{i+1} + R_{i+2}} \quad i = 1,2,\ldots,N$$

$$= 0 \quad i = N \quad (51)$$

$$\gamma_C = V_1^2 R_C^{2m_{i+1}} \quad (52)$$

$$\gamma_U = \frac{U_1}{2} R_N^{m_{i+1}} \quad (53)$$

$$\gamma_p = Pe_1 R_{pe}^{m_{i+1}} \quad (54)$$

$$\gamma_v = R_u^{m_{i+1}} \quad (55)$$

$$Y_1 = \frac{\gamma_C S^2}{\gamma_V^2} + \frac{S(\gamma_p + 2\gamma_U\gamma_C)}{\gamma_V} - \gamma_p\gamma_U(\gamma_2+\gamma_3) - \frac{\gamma_U\gamma_C S}{\gamma_V}(\gamma_2+\gamma_3) + 2\gamma_p\gamma_U \quad (56)$$

$$Y_2 = -\gamma_p\gamma_U\gamma_1 - \frac{\gamma_U\gamma_C\gamma_1 S}{\gamma_V} \quad (57)$$

$$Y_3 = -\gamma_p\gamma_U\gamma_4 - \frac{\gamma_U\gamma_C\gamma_4 S}{\gamma_V} \quad (58)$$

$$Y_4 = (-1)^{i-1}\left[\frac{2S\gamma_C}{\gamma_V} + (\gamma_p + 2\gamma_U\gamma_C) - (\gamma_2+\gamma_3)\gamma_U\gamma_C\right] \quad (59)$$

$$Y_5 = -(-1)^{i-1}\gamma_U\gamma_C\gamma_1 \quad (60)$$

$$Y_6 = -(-1)^{i-1}\gamma_U\gamma_C\gamma_4. \quad (61)$$

Likewise the boundary conditions given by equation (40)–(43) can also be transformed to:

at $x = 0$

$$\left(1 + \frac{V_1^2}{Pe_1}S\right)t_i - \frac{1}{Pe_{C1}}\frac{dt_i}{dx} = F_1(S)\,e^{-\phi_i S}$$

$$\left[i = 1,3,5,\ldots,2\left(\frac{N+1}{2}\right)-1\right] \quad (62)$$

$$\frac{dt_i}{dx} = 0 \quad \left[i = 2,4,6\ldots,2\left(\frac{N}{2}\right)\right] \tag{63}$$

at $x = 1$

$$\left(1+\frac{V_2^2 S}{Pe_2 R_u}\right)t_i + \frac{1}{Pe_{C2}}\frac{dt_i}{dx} = F_2(S)\,e^{-\phi_i S}$$

$$\left[i = 2,4,6,\ldots,2\left(\frac{N}{2}\right)\right] \tag{64}$$

$$\frac{dt_i}{dx} = 0 \quad \left[i = 1,3,5\ldots,2\left(\frac{N+1}{2}\right)-1\right]. \tag{65}$$

The system of transformed equations (47) can now be written in the matrix format as

$$\frac{d\mathbf{T}}{dx} = \mathbf{AT} \tag{66}$$

where the vector $\mathbf{T}$ is given by

$$\mathbf{T} = \left(T_1, T_2, \ldots, T_N, \frac{dT_1}{dx}, \frac{dT_2}{dn}, \ldots \frac{dT_N}{dx}\right)^{\mathrm{T}}. \tag{67}$$

The matrix $\mathbf{A}$ is the coefficient matrix of the set of ordinary differential equations (47) which can be conveniently expressed in terms of Y_1, Y_2, ... Y_6 and Y_C.

The matrix equation (66) can be solved by evaluating eigenvalues β_j and eigenvectors $[g_j]$ of the coefficient matrix $\mathbf{A}$. This boundary value problem has the solution in the form

$$\mathbf{T} = \mathbf{GB}(x)\mathbf{D} \tag{68}$$

where the diagonal matrix $\mathbf{B}(x)$ is given by

$$B(x) = \mathrm{diag}\,\{e^{\beta_1 x}, e^{\beta_2 x}, \ldots, e^{\beta_{2N} x}\}. \tag{69}$$

The columns of matrix $\mathbf{G}$ are the eigenvectors of matrix $\mathbf{A}$, and $\mathbf{D}$ is a coefficient vector which can be evaluated from the boundary conditions. Thus the fluid temperature can be expressed as

$$T_i = \sum_{j=1}^{2N} d_j g_{ij}\, e^{\beta_j x} \tag{70}$$

the derivatives of the temperatures can be expressed as

$$\frac{dT_i}{dx} = \sum_{j=1}^{2N} d_j g_{N+i,j}\, e^{\beta_j x}. \tag{71}$$

Equations (70) and (71) can be applied to the boundary conditions (62) to (65) which gives

$$\mathbf{WD} = \mathbf{F}. \tag{72}$$

The vector $\mathbf{F}$ contains the input functions to the channels and the matrix $\mathbf{W}$ results from boundary conditions (62)–(65)

$$\mathbf{F} = [F_1(S)\,e^{-\phi_1 S}, F_2(S)\,e^{-\phi_2 S}, F_1(S)\,e^{-\phi_3 S},$$
$$F_2(S)\,e^{-\phi_4 S}, \ldots F_K(S)\,e^{-\phi_N S}, 0, 0\ldots]^{\mathrm{T}} \tag{73}$$

where $K = 1$ for N odd

$= 2$ for N even.

Hence from the boundary conditions the coefficient matrix $\mathbf{D}$ can be obtained as

$$\mathbf{D} = \mathbf{W}^{-1}\mathbf{F}. \tag{74}$$

Thus the solution is obtained in the Laplace domain which is again to be reverted back to the time domain by Laplace inversion. Obviously the only way to do it is by numerical inversion. In the present analysis a Fourier series approximation method is used for Laplace inversion which is applicable both to step and sinusoidal responses [17]. The method is accelerated by fast Fourier transformation to evaluate the response at the nth time step for a function $\mathbf{H}$ (S) to the time domain in the form H (Z_n).

$$H(Z_n) = \frac{\exp(aZ_n)}{Z}\left[Re\sum_{k=0}^{M-1}\mathbf{H}\left(a+\frac{ik\pi}{Z}\right)\right.$$
$$\left.\times\exp\left(i\frac{2\pi nk}{M}\right)-\tfrac{1}{2}H(a)\right]. \tag{75}$$

The constant a lies in the domain $4 < aZ < 5$ for minimization of truncation error.

The response is obtained by this numerical inversion for each channel, keeping in mind the phase lag at the exit port. The summation is performed to give the combined fluid temperature at the outlet of the heat exchanger, thus

$$T_{g1,\mathrm{out}}(Z) = \frac{1}{n_1}\sum_{i=1}^{n_1} T_{1,2i-1}(Z-\phi_{2i-1}) \tag{76}$$

$$T_{g2,\mathrm{out}}(Z) = \frac{1}{n_2}\sum_{i=1}^{n_2} T_{2,2i}(Z-\phi_{2i}). \tag{77}$$

RESULTS AND DISCUSSION

With the help of the analysis presented in the preceding sections the temperature response due to any type of temperature transient at one or both of the inlets can be obtained. The two types of temperature transients that are mostly of practical importance, namely, the step and sinusodial change, have been presented here as examples. In these examples the realistic values for heat exchanger parameters such as NTU and R_{g2} have been chosen and they have been kept constant in all the examples since the primary objective of the paper is to investigate the effect of the dispersion characteristic Pe along with its finite propagation velocity C^*. The plate spacing is chosen to be 4% of the effective flow length within channels and the diameters of the circular ports from which the channels are fed and at which the flow reassembles at outlet are chosen in such a way that the Péclet number remains the same in all the channels. However, the method can also be applied where the two sides have different Péclet numbers.

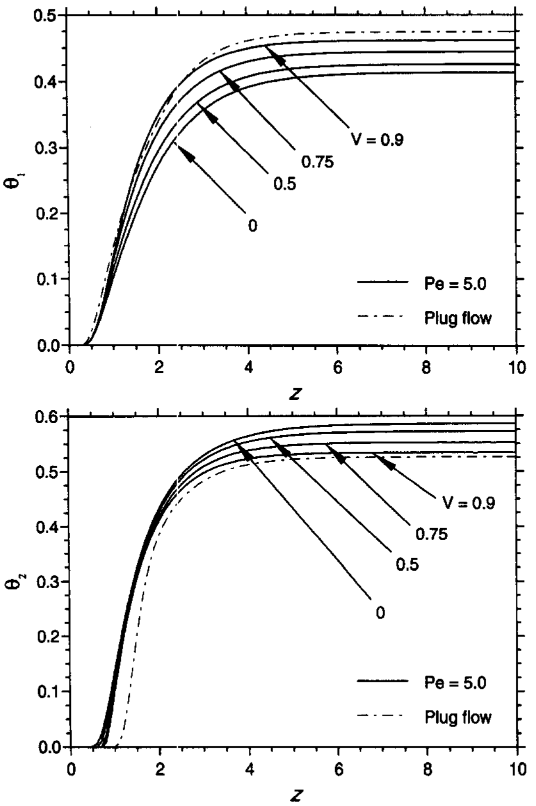

Fig. 2. Effect of finite dispersion wave velocity, at constant Péclet number $Pe = 5.0$ on the exit response of a plate heat exchanger due to a step change of temperature at the inlet of the fluid in side 2. (a) Response of fluid 1. (b) Response of fluid 2.

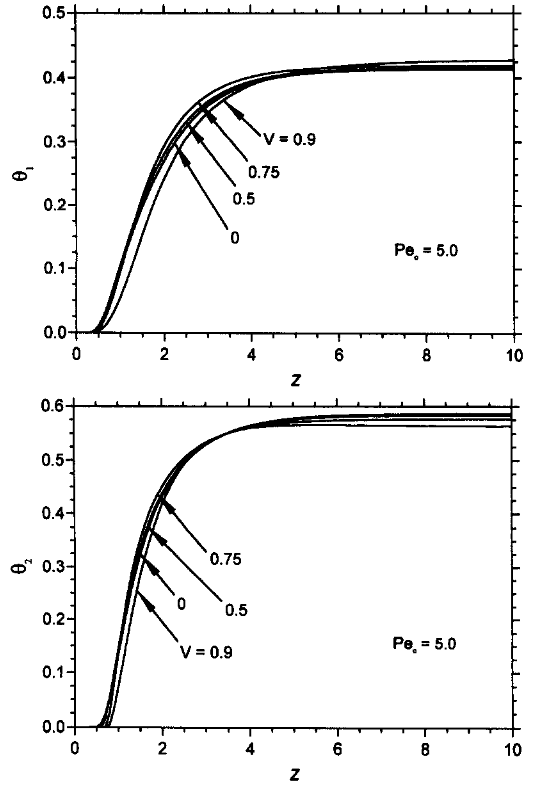

Fig. 3. Effect of finite dispersion wave velocity, at constant effective Péclet number $Pe_C = 5.0$ on the exit response of a plate heat exchanger due to a step change of temperature at the inlet of the fluid in side 2. (a) Response of fluid 1. (b) Response of fluid 2.

With the chosen parameter values $N = 15$, $R_w = 0.4$, $R_\tau = 1.0$, $R_{g2} = 1.0$, $R_N = 1.0$, $NTU_1 = 1.0$ and $Pe = 5.0$ the step responses have been calculated for different values of the ratio V of the fluid and dispersion wave velocity. This depicts the effect of hyperbolic dispersion on the step response. Obviously $V = 0$ corresponds to the traditional parabolic dispersion. The results are presented in Fig. 2. It is observed that for a given Péclet number Pe, the steady-state temperature at the outlet of the cold fluid increases and that of the hot fluid decreases with the increase of V. This implies that the dispersion wave velocity changes the dispersion effect even for a given Péclet number. The higher the ratio V of the fluid to the dispersion wave velocity the lower is the dispersion effect. Theoretically as $V \to 1$ the steady-state response approaches the plug flow model which is also shown in the figure. From the steady-state temperatures it can be concluded that the decrease of dispersion wave velocity C^* increases the heat exchanger effectiveness by decreasing the dispersion effect and when the dispersion wave propagates at the same velocity as the fluid the dispersion effect disappears in the steady state and the heat exchanger behaves as if a plug flow is taking place.

To explain the effect of dispersion wave propagation another approach can be taken in congruence with the observation [10] that at steady state the effective dispersion coefficient can be taken as $\lambda^*(1 - V^2)$. Thus the effective Péclet number can be defined as

$$Pe_C = \frac{Pe}{1 - V^2}.$$

In fact in Fig. 2 the difference of the effectiveness is primarily due to the difference in the effective Péclet number. To demonstrate this the step responses for constant effective Péclet number Pe_C are plotted in Fig. 3. It is observed that at steady state the effective Péclet number approximately depicts the dispersion effect uniquely since the curves approach each other though the difference remains. However, it is interesting to observe that the responses in the transient regime depend on the individual values of V, and the effective Péclet number alone does not represent the true dispersion effect. This difference is reflected as the difference in the slopes of the curves in Fig. 3.

At this point it can be mentioned that while verifying the dispersion effect with experimentation on shell and tube heat exchangers, it was observed that the slope of the experimental dynamic response differs from that calculated with the Péclet number which gives the same steady-state temperature [2]. The pre-

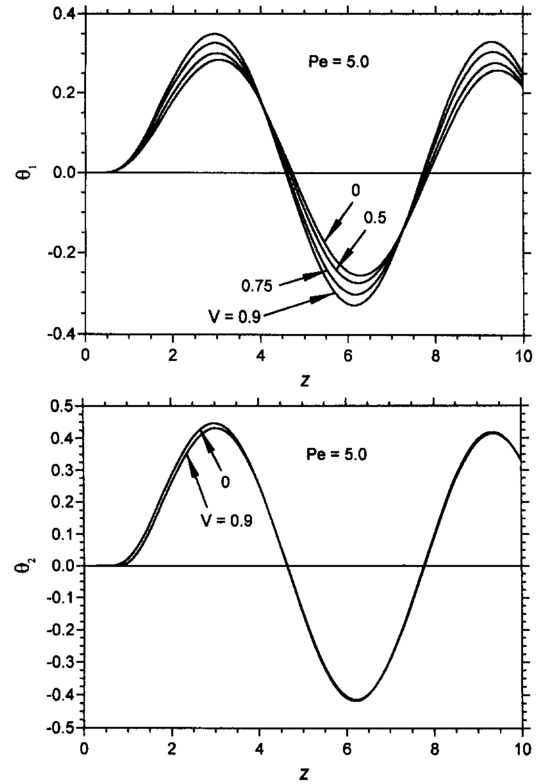

Fig. 4. Effect of finite dispersion wave velocity, at constant Péclet number $Pe = 5.0$ on the exit response of a plate heat exchanger due to a sinusoidal variation of the inlet temperature of side 2. (a) Response of fluid 1. (b) Response of fluid 2.

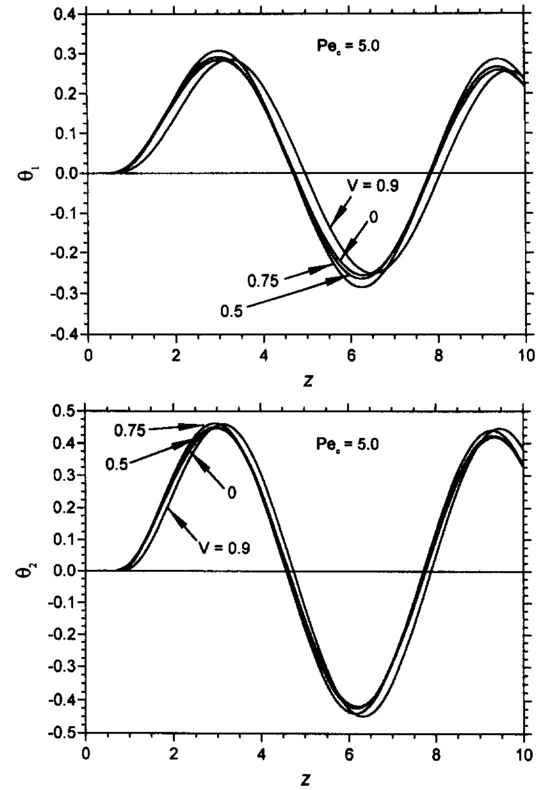

Fig. 5. Effect of finite dispersion wave velocity, at constant effective Péclet number $Pe_C = 5.0$ on the exit response of a plate heat exchanger due to a sinusoidal variation of the inlet temperature of side 2. (a) Response of fluid 1. (b) Response of fluid 2.

sent theory appears to be a good explanation for such behaviours when applied to the respective cases.

The marginal difference in the steady-state response for constant effective Péclet number Pe_C and different dispersion wave velocity, as observed in Fig. 3, can be attributed to the last term of equation (12) where the dispersion properties appear in explicit form.

The response due to sinusoidal oscillation in one of the inlet temperatures brings out some interesting features because it is a strongly transient phenomenon. Figure 4 shows such a response for the same set of values of the constants as used for the step response previously. The hot-side inlet temperature is taken as a pure sine wave of the form

$$T_{2,\text{in}}(Z) = \sin Z. \quad (78)$$

For a constant Péclet number Pe it is observed that the amplitude of the cold-side outlet temperature increases with the increase of V. The case with the hot-side outlet temperature is just the reverse although the effect of V here is not as predominant as in the case of cold fluid. These characteristics as presented in Fig. 4, reconfirm the fact that the dispersion wave velocity controls the effect of dispersion and the lower the value of the velocity C^* the lower is the effect of axial dispersion of the fluid. It is also worth noting that the phase shift of the waves decreases for the cold fluid with increasing value of V, while in the case of the hot fluid it increases. One interesting feature is noted in the responses where it is found that in each half cycle the responses of both the hot and cold sides intersect at a common point for a given value of Pe and different values of V. Figure 5 depicts the sinusoidal response for a given effective Péclet number Pe_C and different ratios of the fluid to dispersion wave velocity V. It is found that the approach of using the effective Péclet number does not bring out any distinct advantage here since the process is totally transient in this case. It is observed that both the hot- and cold-side responses depend strongly on the dispersion wave velocity in respect of amplitude and phase shift.

Finally, responses are computed by doubling the frequency of the inlet temperature sinusoid with the same set of constants as in the previous case. The results are presented in Fig. 6. It is found that for a given Péclet number Pe, the difference in amplitude is higher for various values of V than for lower frequency for both hot- and cold-side temperatures. This is because at higher frequency the heat exchanger encounters stronger temperature transients and the greater the transient behaviour the more is the effect of dispersion wave velocity. The absolute value of phase shift is found to be of the same order at higher frequency but since the period is one half in this case

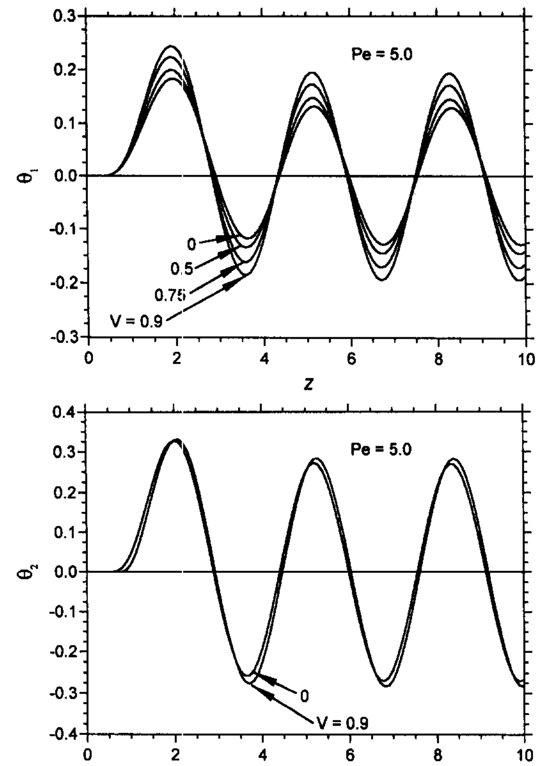

Fig. 6. Effect of dispersion wave velocity, at constant Péclet number $Pe = 5.0$ on the exit response of a plate heat exchanger due to a higher frequency sinusoidal variation of the inlet temperature of side 2. (a) Response of fluid 1. (b) Response of fluid 2.

so the relative phase shift is almost doubled. Finally it is observed once again that even at higher frequency all the curves intersect at a common point during each half period of the cycles.

Since the thrust of the present analysis is to observe the effect of dispersion wave velocity, the effects of other parameters such as NTU or R_2 have not been computed here. They, however, show the expected characteristic.

SUMMARY AND CONCLUSIONS

A new concept of hyperbolic dispersion is introduced in the present paper. Based on the theory of hyperbolic (or the so-called 'non-Fourier') conduction the basic equations for conduction of heat during fluid flow have been derived. Then, by analogy with conduction, the concept of hyperbolic dispersion has been brought to light. This concept is based on the premise that the axial dispersion in fluid, which has been used in previous work as a phenomenon that describes the deviation from ideal plug flow, propagates with a finite wave velocity rather than the instantaneous propagation assumed earlier. It is further observed that for axial dispersion in fluid with a wall heat flux, a delay can also be assumed in the wall heat flux and this model can be termed as a 'regenerator model' [10]. However, due to mathematical difficulties it is difficult to introduce such a delay term for recuperators where the wall is wetted by two fluids simultaneously. In this case the normal convection equation can be used and the model can be termed as a 'general heat exchanger model' [11]. Salient features of these two models are also discussed in the text.

An extended form of the Danckwert [16] boundary condition at an inlet is proposed for the present model which arises from the energy balance at the heat exchanger inlet cross-section, taking the finite dispersion wave velocity into consideration.

Using the 'general heat exchanger model', a U-type plate heat exchanger is modelled for transient response. In the present model longitudinal conduction through the plates is neglected. With a dispersive Péclet number depicting the axial dispersion in the fluid and dispersion wave velocity depicting the propagation of this dispersion, the model is formulated in the form of a set of partial differential equations in time and space coordinates. This set of equations is solved by the method of Laplace transforms and is then inverted back to the real time domain by the numerical inversion of the Laplace transform using fast Fourier transforms (FFT).

It is found that the dispersion wave velocity plays a significant role particularly in the transient regime. The slopes and the steady-state values of the temperature response depend on this wave velocity. It is found that, even though in the steady state the normal dispersion method with a corrected effective Péclet number (infinite dispersion wave velocity) can be used as a rough approximation, it fails severely in the transient domain of the response. Strong effect of dispersion wave velocity has also been observed for oscillatory responses. In this case also the decrease of dispersion wave velocity decreases in the axial dispersion phenomenon of the fluid. The use of effective Péclet number does not appear to be meaningful in such strongly transient cases. Finally it is observed that at higher frequency the dispersion wave velocity plays an even bigger role in respect of the amplitude and phase shift of the response caused by an increased transient nature with the increase of frequency. The results clearly indicate the utility of using the proposed method for heat exchanger analysis for which this paper may act as an introduction. However, the following further investigations are suggested to develop it as a fully fledged instrument for heat exchanger analysis:

(1) Based on the analysis presented earlier [10, 11] and here, transient test techniques should be devised for the determination of dispersive Péclet number and dispersion wave velocity for different flow conditions.
(2) Transient experiments on heat exchangers should be performed on different types of heat exchangers and the responses are to be analysed by the present analysis.

(3) The 'regenerator model' can be used to analyse the storage-type heat exchangers.

(4) Detailed numerical calculations based on actual flow patterns should be carried out to assess the goodness of the analysis presented here.

REFERENCES

1. B. Spang, Über das thermische Verhalten von Rohrbündelwärmeübertragern mit Segmentumlenkblechen. Reihe 19, Nr. 48. VDI Verlag (1991).
2. W. Roetzel, Transient analysis in heat exchangers, presented at the ICHMT International Symposium on New Developments in Heat Exchangers, Lisbon, Portugal, September 6–9 (1993).
3. W. Roetzel and Y. Xuan, Analysis of transient behaviour of multipass shell and tube heat exchangers with the dispersion model, *Int. J. Heat Mass Transfer* **35**, 2953–2962 (1992).
4. Y. Xuan and W. Roetzel, Stationary and dynamic simulation of multipass shell and tube heat exchangers with the dispersion model for both fluids, *Int. J. Heat Mass Transfer* **36**, 4221–4231 (1993).
5. P. M. Morse and H. Feshbach, *Methods of Theoretical Physics*, p. 865, McGraw-Hill, New York (1953).
6. E. R. G. Eckert and R. M. Drake Jr, *Analysis of Heat and Mass Transfer*, p. 23, McGraw-Hill, Tokyo (1972).
7. M. Chester, Second sound in solids. *Phys. Rev.* **131**, 2013–2015 (1963).
8. J. I. Frankel, B. Vick and M. N. Ozisik, Flux formulation of hyperbolic heat conduction, *J. Appl. Phys.* **58**, 3340 (1985).
9. R. K. Sahoo, Propagation of thermal waves with laterial heat transfer, *Cryogenics* **34**, 203 (1994).
10. W. Roetzel and B. Spang, Axiales Dispersionmodell für Regeneratoren mit endlicher Ausbreitungsgeschwindigkeit thermischer Störungen, Mitteilung aus dem Institut für Thermodynamik, No. 1, Universität der Bundeswehr Hamburg (1994).
11. W. Roetzel and B. Spang, Axiales Dispersionsmodell für Wärmeübertrager mit endlicher Ausbreitungsgeschwindigkeit thermischer Störungen, *Uniforschung* 2, Universität der Bundeswehr Hamburg, pp. 37–47 (1995).
12. A. R. Khan, N. S. Baker and A. P. Wardley, The dynamic characteristics of a countercurrent plate heat exchanger, *Int. J. Heat Mass Transfer* **31**, 1269–1278 (1988).
13. C. C. Lakshmann and O. E. Potter, Dynamic simulation of plate heat exchangers, *Int. J. Heat Mass Transfer* **33**, 995–1002 (1990).
14. S. K. Das and W. Roetzel, Dynamic analysis of plate heat exchangers with dispersion in both fluids, *Int. J. Heat and Mass Transfer* **38**, 1127–1140 (1995).
15. S. K. Das, B. Spang and W. Roetzel, Dynamic behaviour of plate heat exchangers—experiments and modelling, *J. Heat Transfer*, *Trans. ASME* (in preparation).
16. P. V. Danckwert, Continuous flow systems—distribution of residence times, *Chem. Engng Sci.* **2**, 1–13 (1953).
17. K. S. Crump, Numerical inversion of Laplace transforms using a Fourier series approximation, *J. Assoc. Comput. Mach.* **23**, 89–96 (1976).